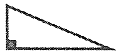
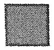
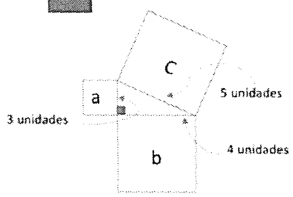
Teorema de Pitágoras

Definición del Teorema de Pitágoras

1. Si tienes un triángulo rectángulo



2. Haces un  (cuadrado) en cada lado

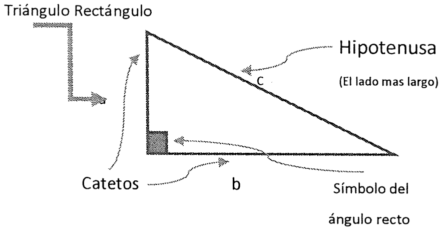


3. Entonces el cuadrado mayor, c, tiene la misma área que los otros dos, a y b, juntos.

$a^2 + b^2 = c^2$

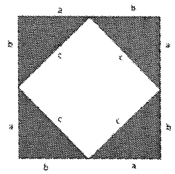
$$a^2 + b^2 = c^2$$

Vocabulario



Prueba

- Usando datos conocidos para probar algo en cada paso.



1) El área de un [Cuadrado grande] es:

A□= (a+b)(a+b)

2) Área de las partes mas pequeñas:

□ pequeño A□ = C^2

Triángulos $A_\Delta = \frac{1}{2}(ab)$

los 4 $A_\Delta = 4(\frac{1}{2}ab)$

$A_\Delta = 2ab$

Área total de las partes:

$A_{Total} = c^2 + 2ab$

Conclusión

Las dos áreas deben ser IGUALES

$(a+b)(a+b) = c^2 + 2ab$

$a^2 + \cancel{2ab} + b^2 = c^2 + \cancel{2ab}$

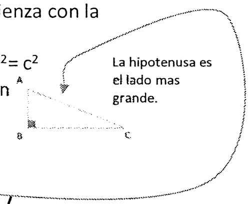
$a^2 + b^2 = c^2$

Prueba Inversa

- Una prueba que comienza con la conclusión.

Dado: ΔABC con $a^2+b^2= c^2$

Prueba que ΔABC es un



Lados:

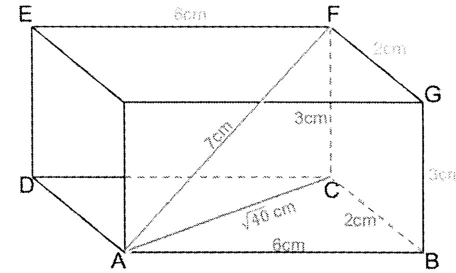
6 cm	$6^2+ 8^2 = 10^2$
10 cm	36 + 64 =100
8 cm	100 = 100. ✓

¿Cómo seria un △ agudo?
Si $c^2 < a^2+ b^2$

¿Cómo seria un △ obtuso?
Si $c^2 > a^2+ b^2$

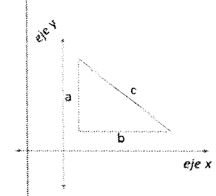
¿Por qué?

Encontrar un lado desconocido en figuras de 2 dimensiones y figuras de 3 dimensiones



Encontrar el lado AF usando el Teorema de Pitágoras.

1. ΔABC	2. ΔACF
$AC^2 = 6^2 + 2^2$	$AF^2 = AC^2 + CF^2$
$AC^2=36+4$	$AF^2= 40 + 3^2$
$AC^2=40$	$AF^2=49$
$AC= \sqrt{40}$	$AF= \sqrt{49}$



Usando el sistema de coordenadas

A (2, 10)	$c^2 = a^2+ b^2$
B (8,2)	$c^2 = 8^2 + 6^2$
Encuentra el lado c	$c^2 = 64 + 36$
(hipotenusa)	$c^2 = 100$
	$c = 10$

Estándares para la práctica matemática

- Poner atención a la precisión cuando se usa la terminología para referirse a figuras geométricas.
- Construye argumentos viables y critica el razonamiento de otros cuando expliquen una prueba del Teorema de Pitágoras y su prueba inversa.

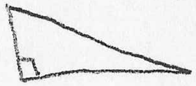
Investigación: "Teorema de Pitágoras"

Que sabemos... Que queremos saber..

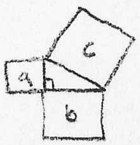
Pythagorean Theorem

Pythagorean Theorem Defined

① If you have a right triangle



② And you make a □ on each side

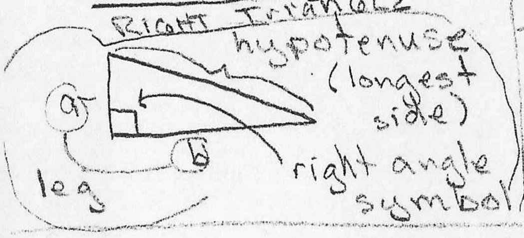


③ then the biggest [c] has the same area as the two smaller [a] [b] put together

[a^2] + [b^2] = [c^2]

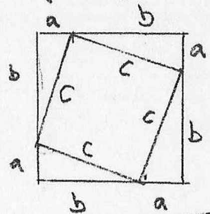
$a^2 + b^2 = c^2$

Vocabulary



Proof

· using known facts to prove something in steps



① Area of [large square] is:

$A = (a+b)(a+b)$

② Areas of smaller pieces:

Small □ $A_{\square} = c^2$

triangles △ $A_{\triangle} = \frac{1}{2}ab$

· all 4 △s $A_{\triangle} = 4(\frac{1}{2}ab)$

$= 2ab$

Total area of parts:

$A_T = c^2 + 2ab$

Conclusion

Both areas must be equal

$(a+b)(a+b) = c^2 + 2ab$

$a^2 + \cancel{2ab} + b^2 = c^2 + \cancel{2ab}$

$\boxed{a^2 + b^2 = c^2}$

DONE!

Converse Proof

· a proof that starts with the conclusion

Given: △ABC with

$a^2 + b^2 = c^2$

prove △ABC is a right △

6cm 10cm 8cm

$6^2 + 8^2 = 10^2$

$36 + 64 = 100$

$100 = 100$ ✓

what would make an acute △?

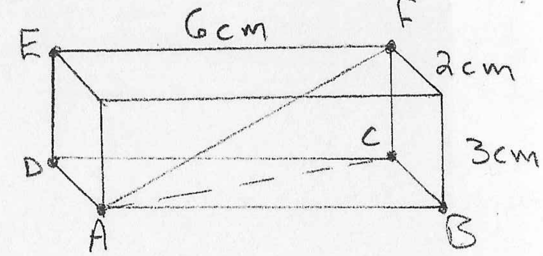
If $c^2 < a^2 + b^2$

Obtuse △?

If $c^2 > a^2 + b^2$

[corollary to the converse]

Finding an unknown length in 2 Dimensional & 3D figures



Find length of AF using Pythagorean Theorem

① △ABC

$AC^2 = 6^2 + 2^2$

$AC^2 = 36 + 4$

$AC^2 = 40$

$AC = \sqrt{40}$

② △ACF

$AF^2 = AC^2 + CF^2$

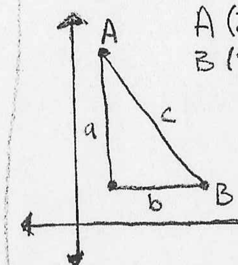
$AF^2 = 40 + 3^2$

$AF^2 = 49$

$AF = \sqrt{49}$

$AF = \boxed{7\text{ cm}}$

Using the coordinate system



A (2,10)
B (8,2)

$c^2 = a^2 + b^2$

$c^2 = 8^2 + 6^2$

$c^2 = 64 + 36$

$c^2 = 100$

$c = 10$

Mathematical Standards and Practices

- We will attend to precision when using terminology to refer to geometric figures.
- We will construct viable arguments and critique the reasoning of others when explaining a proof of the Pythagorean theorem and its converse.

Inquiry Pythagorean Theorem

We know...

We want to know...

AIM4S³ ™

UNIT PLANNING TOOL

Unit 8th: Geometry

CCSSM: 8.G.B.6

- Explain a proof of Pythagorean Theorem and its Converse

8.G.B.7 • Apply Pythagorean Theorem to determine unknow side lengths in right triangles in real and mathematical problems with 2D & 3D figures

8.G.B.8 • Apply Pythagorean Theorem to find distances between points on coordinate systems

Math Practices being emphasized:

#3 Construct viable arguements and critique the reasoning of others

#6 Attend to precision

~~Essential Questions~~

Essential Questions

How can Pythagorean Theorem be used to solve problems?

How do I know that I have a convincing arguement to informally prove PT?

Pre and Post Assessments

Pre

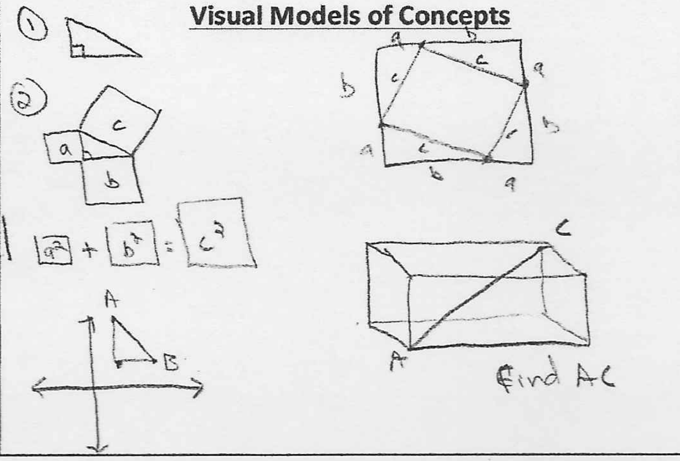
Teacher Made
- see attached

Post
- unit assessment
- Performance Task

Key Concepts

- Coordinates can be used to measure distances
- Right triangles have a special relationship among the side lengths which can be represented by a model and a formula
- Recognizing Pythagorean Theorem Triples can increase efficiency with problems involving PT

Visual Models of Concepts



Algorithms/Diagrams

$a^2+b^2=c^2$

$A_{\square}=(a+b)(a+b)$

$A_{\triangle}=\frac{1}{2}(ab)$

$\times 4 \quad = 4(\frac{1}{2})ab$

$= 2ab$

$A_{\square}=c^2$

$(a+b)(a+b)=c^2+2ab$

if $c^2<a^2+b^2$ then acute $\triangle$

if $c^2>a^2+b^2$ then obtuse $\triangle$

Connections (Real World Applications)

- Be able to find an unknown length
- Be able to find a difficult length to measure
- Carpentry
- Engineers
- Architects
- Construction
- Sculpture
- Art

Language Functions/Structures

Explain

If you have ______ then you can determine ______.

Compare

This area must be equal to that area because ______.

Vocabulary

Altitude of triangle
Base (of a polygon)
Coordinate Plane
Converse of Pythagorean Theorem
Cubic Root
Hypotenuse
Leg of a triangle
Perfect Squares
Perfect cubes
Pythagorean theorem
Pythagorean Triples
Square Root
Right Triangle

Focus and Motivation

Literature

What's your Angle, Pythagoras?
by Julie Ellis

Animation

Brainpop
- Pythagorean theorem

Tools of Trade

- right angle
- framing square
- Sliding T-bevel
- Level

Art

Pythagorean Spiral

Pythagorean Tree

PYTHAGOREAN THEOREM

Pythagorean Theorem Defined

① If you have a right triangle

② And you make a square on each side

③ then the biggest has the same area as the two smaller a b put together

$a^2 + b^2 = c^2$

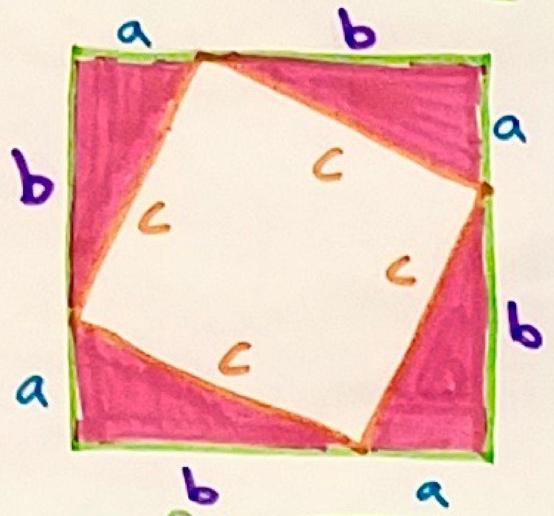
3 units, 4 units, 5 units

Vocabulary

Right Triangle, hypotenuse (longest side), Legs a, b, Right angle symbol

Proof

- using known facts to prove something in steps



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② Area of smaller pieces:

Small □ $A_{\square} = c^2$

Triangles $A_{\Delta} = \frac{1}{2}(ab)$

All 4 $A_{\Delta} = 4(\frac{1}{2}ab)$

$A_{\Delta} = 2ab$

Total Area of parts:

$A_{Total} = c^2 + 2ab$

Conclusion

Both areas must be EQUAL

$(a+b)(a+b) = c^2 + 2ab$

$a^2 + \cancel{2ab} + b^2 = c^2 + \cancel{2ab}$

$a^2 + b^2 = c^2$ DONE!

Converse Proof

- a proof that starts with the conclusion

GIVEN: ΔABC with $a^2 + b^2 = c^2$

Prove ΔABC is a right triangle (hypotenuse is the longest side)

Sides

6 cm, 10 cm, 8 cm

$6^2 + 8^2 = 10^2$

$36 + 64 = 100$

$100 = 100$ ✓

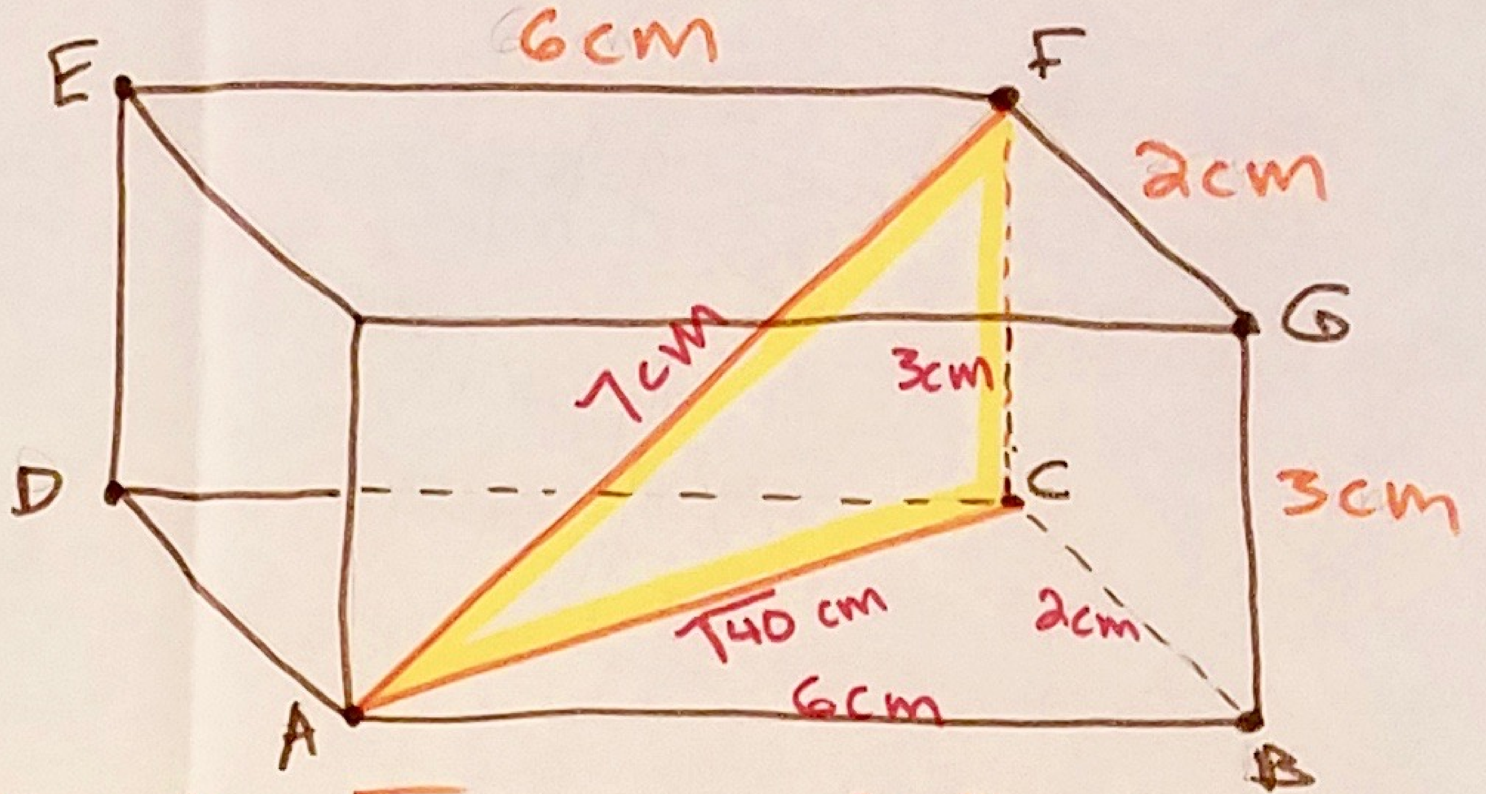
What would make an acute Δ? WHY?

If $c^2 < a^2 + b^2$

What would make an obtuse Δ? WHY?

If $c^2 > a^2 + b^2$

Finding an unknown length in 2Dimensional and 3D figures



Find length $\overline{AF}$ using Pythagorean Theorem

① ΔABC

$AC^2 = 6^2 + 2^2$

$AC^2 = 36 + 4$

$AC^2 = 40$

$AC = \sqrt{40}$

② ΔACF

$AF^2 = AC^2 + CF^2$

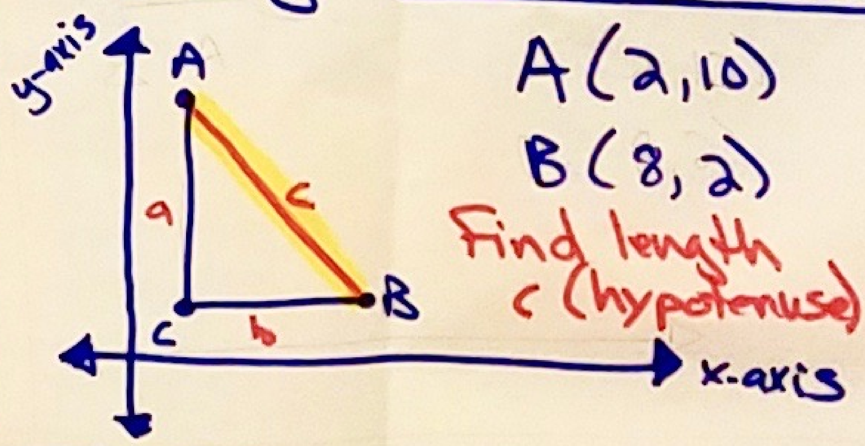
$AF^2 = 40 + 3^2$

$AF^2 = 49$

$AF = \sqrt{49}$

$AF = 7$ cm

Using the Coordinate System



A(2,10)

B(8,2)

Find length c (hypotenuse)

$c^2 = a^2 + b^2$

$c^2 = 8^2 + 6^2$

$c^2 = 64 + 36$

$c^2 = 100$

$c = 10$

Mathematical Standards & Practices

- We will attend to precision (be correct) when using terminology (vocabulary) to refer to geometric (shapes) figures.
- We will construct viable (on topic) arguements (debates) and critique (check accuracy) the reasoning of others when explaining a proof of the Pythagorean Theorem and its converse (a proof that starts with the conclusion).

We know...

It has to do with angles.

It is specific to the Right triangle

It has to do with right angles.

It has to do with Δ.

The legs are ~~mostly~~ the sides ~~with~~ of right ~~angles~~ triangles (the sides).

? Inquiry ?

Pythagorean Theorem

We want to know...

What is the Pythagorean theorem? When each □ on the legs of the right triangle are equal to the square on the hypotenuse we Pythagorean Theorem (activity).

Why do we need it in our every day life? measure - tile - stairs - art

How to use it? We can find an unknown. We can find a measurement (lesson) that is hard to physically measure. (experience)