**Real Numbers**

- **Rational Numbers**
  - $\frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$
  - Integers: $-3, 1, 0, 1, 2, 3, \ldots$
  - Whole numbers: $0, 1, 2, 3, \ldots$
  - Natural numbers: $1, 2, 3, \ldots$
  - $\sqrt{2}, \sqrt{3}, \sqrt{5}$

- **Irrational Numbers**
  - $\sqrt{2}, \sqrt{3}, \sqrt{5}$
  - $\pi$, $\phi$ (Golden Ratio)

**Scientific Notation**

<table>
<thead>
<tr>
<th>Scientific Notation</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 10^4$</td>
<td>$30000$</td>
</tr>
<tr>
<td>$3.4 \times 10^{-5}$</td>
<td></td>
</tr>
</tbody>
</table>

**Exponents**

- **Exponential Form**
  - $a^b = c$
  - $b$ is the exponent, $a$ is the base, $c$ is the result.

**Properties of Exponents**

- $a^0 = 1$
- $a^{-n} = \frac{1}{a^n}$
- $(a^m)^n = a^{mn}$
- $a^{m/n} = \sqrt[n]{a^m}$

**Real Numbers, Scientific Notation and Exponents**

- To write a number in scientific notation, the number multiplied by 10 must be less than 10 and closer to one.

**Expanded Form**

- $3 \times 10^3 + 4 \times 10^2 + 2 \times 10^1 + 6 \times 10^0 + 4 \times 10^{-1} + 3 \times 10^{-2}$

**Integers**

<table>
<thead>
<tr>
<th>Model</th>
<th>Proof</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^3 \times 2^5 = 2^8$</td>
<td>$(2^3)^3 = 2^9$</td>
<td>$a^m \times a^n = a^{m+n}$</td>
</tr>
<tr>
<td>$3^2 \times 3^3 = 3^5$</td>
<td>$3^2 \times 3^3 = 3^5$</td>
<td>$(a^m)^n = a^{mn}$</td>
</tr>
</tbody>
</table>

**Standards and Mathematical Practices**

- We will evaluate square roots of small perfect squares and cube roots of small perfect cubes by looking for and making use of structure.
- We will attend to precision in knowing rational and irrational numbers.
- We will reason abstractly and quantitatively when using scientific notation and choosing units of appropriate size for measurements of very large or very small quantities.
Calculus is the theory of change. It is necessary for numbers and rational/irrational numbers and

Essential Question(s):

- Exponents are useful for representing very large/small numbers.

Pre and Post Assessments

Give 2 examples of a rational number. Write 3,000,000 and 0.000047

Give examples of an irrational number. Simplify $\sqrt{2} \times 6^3$

Rational Numbers

- Cannot be placed on the number line
- $\frac{a}{b}, b \neq 0$

Irrational Numbers

- $\sqrt{2}, \pi, e$
- Cannot be expressed as a fraction, any decimal, or a square root

Key Concepts

- Exponents
- Square roots
- Perfect squares
- Rational numbers
- Irrational numbers
- Rational/irrational numbers
- Real numbers
- Rational/irrational numbers
- Real numbers
- Rational numbers
- Irrational numbers
- $\phi$
- $\pi$
- $e$
- Euler's constant

Visual Models of Concepts

- Square root
- Perfect squares
- Irrational numbers
- Euler's constant
- $\phi$
- $\pi$
- $e$

Connections (Real World Applications)

- Euclid VI.9
- Exponential of an irrational was first realized by Archimedes in the 13th century, etc.
**Language Functions/Structures**

**Vocabulary**
- counting numbers
- whole numbers
- integers
- square roots
- square root
- rational numbers
- irrational numbers
- radical

**Focus and Motivation**
- Brain POP: Way back then
  "Did you know?"
  "Quirky stuff."
- Video: Powers of 10
  (youtube.com/watch?v=OfK5hVdjuo)

**Definition**
Rational numbers are numbers that can be expressed as a ratio of two integers. They can be written as fractions or mixed numbers, and their decimal representations either terminate or repeat. Rational numbers include all integers, fractions, and terminating or repeating decimals.

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**Algebra**

Relationship between numbers

Calculus

Rate of change between numbers

**Visual Models of Concepts**

\[ \sqrt{2} \text{ and } \sqrt{5} = 53 \]

\[ \left( \frac{4}{3} \right)^{2} = \left( \frac{16}{9} \right) = \frac{4}{3}, \text{ and } \sqrt{16} = 4 \]

\[ x^2 = 9 \]

\[ x = \pm 3 \]

\[ \sqrt{19} \]

\[ \text{Real Squares} \]

\[ \sqrt{3} = 1.732050807568877 \]

\[ \sqrt{2} \approx 1.414213562373095 \]

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\[ \sqrt{2} \approx 1.414213562373095 \]

**Connections (Real World Applications)**

- Estimate value of irrational number
  - \( \sqrt{2} \approx 1.41 \)
  - \( \sqrt{3} \approx 1.73 \)
  - \( \sqrt{5} \approx 2.24 \)
  - \( \sqrt{7} \approx 2.65 \)
  - \( \sqrt{11} \approx 3.32 \)
  - \( \sqrt{13} \approx 3.60 \)

- To estimate the square, divide \( \frac{2}{1} \) or \( \frac{1}{2} \)
Scientific Notation

\[ \frac{4^3}{8^2} = \frac{64}{8^2} \]
\[ \frac{4^3}{4^2} = 4^{-1} = \frac{1}{4^1} = \frac{1}{32} \]

Less than 10 and 2 to one

\[ \frac{1}{4^2} \cdot 9^7 = 3.325 = 60 \]
\[ ax^3 + bx + c \]
\[ a(x^3 + b) \]

Integer exponent properties - derive through experience & reason

\[ 2^3 \cdot 3^3 = 2 \cdot 3 \]
\[ 2^3 \cdot 3^3 = 6 \]
\[ 2^3 \cdot 3^3 = 6 \]
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Lesson

Find the approximate square roots by iterative processes.

\[ \sqrt{5} \text{ approximate to the nearest hundredth.} \]
\[ \sqrt{5} \text{ is between 2 and 3.} \]

\[ \sqrt{5} = \sqrt{5.29} \approx 2.29 \]

\[ \sqrt{6} = \sqrt{6.25} \approx 2.5 \]

\[ \sqrt{7} = \sqrt{7.84} \approx 2.8 \]

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### Real Numbers

- **Rational Numbers**
  - Integers: -2, -1, 0, 1, 2, ...
  - Whole Numbers: 0, 1, 2, 3, 4, 5, ...
  - Natural Numbers: 1, 2, 3, 4, ...

- **Irrational Numbers**
  - $\sqrt{2} 
  - \sqrt{5} 
  - \pi 
  - e (Euler's constant)

### Scientific Notation

Scientific notation is used to show very large and small numbers.

<table>
<thead>
<tr>
<th>1000</th>
<th>100</th>
<th>10</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^3</td>
<td>10^2</td>
<td>10^1</td>
<td>10^-1</td>
<td>10^-2</td>
<td>10^-3</td>
</tr>
</tbody>
</table>

- **Examples**:
  - $3.426 \times 10^4$
  - $7.3 \times 10^6$
  - $0.00045$

To write a number in scientific notation, the number multiplied by 10 must be less than 10 and greater than or equal to one.

### Exponents

#### Standard Form
- **Model**
  - $2^3 \cdot 2^5 = 2^8$
  - $(3^2)^3 = 3^6$

#### Proof
- **$x^a \cdot x^b = x^{a+b}$**
  - $2^3 \cdot 2^5 = 2^{3+5} = 2^8$
  - $(3^2)^3 = (3^2)^3 = 3^{2\times3} = 3^6$

#### Rule
- **$x^0 = 1$**
  - $5^0 = 1$
  - $6^{-2} = \frac{1}{6^2}$