Real Numbers, Scientific Notation and Exponents

Real Numbers

Rational

Rational
$3\frac{1}{2}$ $\sqrt{4}$ 3^{-2}

Integers
-2, -1, 0, 1, 2

Whole #'s
0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Natural #'s
1, 2, 3, ...

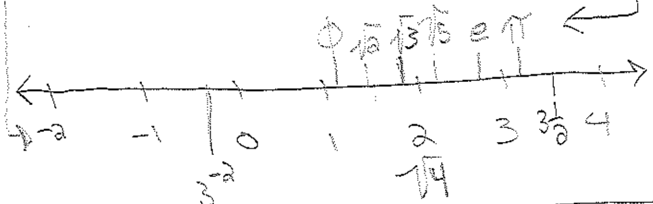
Numbers that can be exactly named, like on a number line

Irrational

ellipsis

ϕ - Phi - Golden Mean $\phi \approx 1.16903...$
$\sqrt{2} \approx 1.41421...$
$\sqrt{3} \approx 1.73205...$
$\sqrt{5} \approx 2.23606...$
e - Euler's Constant $e \approx 2.71828...$
$\pi \approx 3.14159...$

Numbers that cannot be exactly named, they are approximated on the number line



Scientific Notation

used to show large/small numbers

1000	100	10	1		.1	.01
10^3	10^2	10^1	10^0		10^{-1}	10^{-2}
3	4	2	6	.	4	5

To write a number in scientific notation, the number multiplied by 10 must be less than 10 and ≥ to one.

Standard form		Scientific Notation
6,300,000	→	6.3×10^6
.0000067	→	6.7×10^{-6}
730,000,000	←	7.3×10^8
.000045	←	4.5×10^{-5}

Expanded Form

$\boxed{3426.45}$

$3 \times 10^3 + 4 \times 10^2 + 2 \times 10^1 + 6 \times 10^0 + 4 \times 10^{-1} + 5 \times 10^{-2}$

We know...	Inquiry Real #'s Sci. Notation & exponents	We want to know...

Exponents

Exponential Forms

Perfect Squares x^2	Square Roots $\sqrt{x}$	Perfect Cubes x^3	Cubed Roots $\sqrt[3]{x}$	⋮	⋮
$1^2=1$	$\sqrt{1}=1$	$1^3=1$	$\sqrt[3]{1}=1$	5^4	625
$2^2=4$	$\sqrt{4}=2$	$2^3=8$	$\sqrt[3]{8}=2$	5^3	125
$3^2=9$	$\sqrt{9}=3$	$3^3=27$	$\sqrt[3]{27}=3$	5^2	25
$4^2=16$	$\sqrt{16}=4$	$4^3=64$	$\sqrt[3]{64}=4$	5^1	5
$5^2=25$	$\sqrt{25}=5$	$5^3=125$	$\sqrt[3]{125}=5$	5^0	1
$6^2=36$	$\sqrt{36}=6$	$6^3=216$	$\sqrt[3]{216}=6$	5^{-1}	$\frac{1}{5}$
$7^2=49$	$\sqrt{49}=7$	$7^3=343$	$\sqrt[3]{343}=7$	5^{-2}	$\frac{1}{25}$
$8^2=64$	$\sqrt{64}=8$	$8^3=512$	$\sqrt[3]{512}=8$	5^{-3}	$\frac{1}{125}$
$9^2=81$	$\sqrt{81}=9$	$9^3=729$	$\sqrt[3]{729}=9$	5^{-4}	$\frac{1}{625}$
$10^2=100$	$\sqrt{100}=10$	$10^3=1000$	$\sqrt[3]{1000}=10$	⋮	⋮

Integer Exponent Properties

Model	Proof	Rule
$2^3 \cdot 2^5 = 2^8$		
$(3^2)^3 = 3^6$		
$2^3 \cdot 3^3 = 6^3$		
$5^0 = 1$		
$6^{-2} = \frac{1}{6^2}$		

Standards & Mathematical Practices

- We will evaluate square roots of small perfect squares and cubed roots of small perfect cubes by looking for and making use of structure.
- We will attend to precision in knowing rational and irrational numbers.
- We will reason abstractly and quantitatively when using scientific notation and choosing units of appropriate size for measurement of very large/small quantities.

AIM4S³ ™ 8th

calculus is the study of functions of a real variable

Calculus is the theory of measurement. the necessary numbers are rationals & irrationals

UNIT PLANNING TOOL

Unit 1 **:** Real Numbers, Scientific Notation, Exponents

CCSSM: Work w/radicals + integer exponents

8.EE 1, 2, 3, 4, 7
8.NS. 1, 2

MP 2, 6, 7

Essential Question(s):

- Square roots can be rational & irrational
- All real numbers can be plotted on a number line
- Exponents are useful for representing very large/small numbers.

Pre and Post Assessments

Give 2 examples of a rational number.
Give 2 examples of a irrational number.

Write 5,300000 and .000047 in scientific notation

simplify $6^2 \cdot 6^3$

Key Concepts

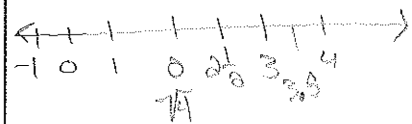
rational #
→ can be placed on the number line

counting #'s
1, 2, 3, 4
whole #'s
0, 1, 2, 3...
integers
0, ±1, ±2, ±3
square (perfect squares)
1, 4, 9, 16, 25, 36

irrational #'s
$\sqrt{2}, \sqrt{3}, \pi$

not possible to name any whole #, any fraction, or any decimal

Visual Models of Concepts



square root

e - Euler's constant
Swiss (Leonhard Euler)

radical sign → $\sqrt{4}$ ← radicand $= 2$

not complete
$\sqrt{2} = 1.41421356237...$
inexact
$\frac{1}{11} = .090909...$ ellipsis

exact
$\frac{1}{4} = .25$

Algorithms/Diagrams

As a fraction $\frac{a}{b}$, where a and b are integers ($b \neq 0$)

Real Numbers

All real numbers are either rational or irrational

Rational
Integers
Whole
Natural

Irrational
Phi
e - Euler's constant

Connections (Real World Applications)

Euclid VI, 9

existence of irrationals was first realized by Pythagoras in the 6th century B.C.

Language Functions/Structures

Vocabulary

rational
is a nameable number,
can be named in a standard way
whole #'s
fractions
mixed #'s
decimal

iterative process
cycle that leads to a closer & closer the desired result

counting #'s
whole #'s
integers
square
square root
radical

radicand
real #'s
radical

Focus and Motivation

square root calculator

Animation

BrainPOP
- rational & irrational numbers
- standard & scientific notation
- exponents
- x & ÷ exponents
- square roots

Literature

BrainPOP "Way back then"
"Did you know"
"Quirky stuff"

Video

Powers of 10
(youtube.com/watch?v=0fKBhvDjuy0

Decimal fractions invented in the 16th Century

Rational # is a # we can know and name exactly, either as a whole #, a fraction, or a mixed number, but not always exactly as a decimal. An irrational number we can never know exactly in any form.

- rational number is of the form $\frac{a}{b}$ where a and b are both integers and b is not 0.
- rational numbers as decimals either repeats or terminates
- irrational numbers as decimals do not repeat or terminate

AIM4S³ ™ 8th CCSS

UNIT PLANNING TOOL

Unit 1 **:** Real Numbers, Scientific Notation, & Exponents

CCSSM:

Essential Question(s):

Numbers and Number systems

Progression of the conception of numbers

counting 1, 2, 3
↓
whole number 0, 1, 2, 3
↓
fractions
↓
decimals
↓
negative fractions
↓
rational numbers
↓
irrational numbers
↓
real numbers
↓
imaginary → complex

Algebra
(relationship between numbers

calculus
(rate of change between numbers

Visual Models of Concepts

$3^2 = 9$ and $\sqrt{9} = \pm 3$

$\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$, and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$

$x^2 = 9$

$\sqrt{x^2} = \pm\sqrt{9}$

$x = \pm 3$

Perfect Squares

1
4
9
16
25
36
49
64
81
100

$\sqrt{1} = 1$

$\sqrt{2} = 1.4142135623730950$

$\sqrt{3} = 1.7320508075688772$

$\sqrt{4} = 2$

$\sqrt{5} = 2.2360679774997$

$\sqrt{6} = 2.449489742783178$

$\sqrt{7} = 2.6457513110645791$

$\sqrt{8} = 2.8284271247461619$

$\sqrt{9} = 3$

Where does $\sqrt{2}$ go among rational numbers

Algorithms/Diagrams

x	x^2
1.0	1.00
1.1	1.21
1.2	1.44
1.3	1.69
1.4	1.96
1.5	2.25

x	x^2
1.40	1.9600
1.41	1.9881
1.42	2.0164

x	x^2
1.410	1.98810
1.411	1.990921
1.412	1.993744
1.413	1.996569
1.414	1.999396
1.415	2.002225

$\sqrt{3}$

x	x^2

Connections (Real World Applications)

estimate value of irrational numbers squares, estimate

ex: 22 → 25 5
  ↘ 36 6

To estimate the square divide $\frac{3}{11} \frac{(25-22)}{(25-36)} = (0.27)$ (5.27)

Scientific Notation

thousands	hundreds	tens	ones		tenths	hundredths	thousandths
1000	100	10	1		.1	.01	.001
10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}
3	2	4	7	.	5	6	8

3247.568

$3\times10^3 + 2\times10^2 + 4\times10^1$
$+ 7\times10^0 + 5\times10^{-1} + 6\times10^{-2}$
$+ 8\times10^{-3}$

$\frac{4^3}{5^2} = \frac{64}{25}$ } $\frac{4^3}{4^7} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$

• less than 10 and ≥ to one

$\frac{1}{4^7} = 4^{-7}$ $\quad 2\cdot3\cdot2\cdot5 = 60$

$2(3)+5$

$2(3+5)$

integer exponent properties — derive through experience & reason

$2^3 \cdot 2^5 = 2^{3+5} = 2^8$

$(2\cdot2\cdot2)\cdot(2\cdot2\cdot2\cdot2\cdot2)$

$3 + 5 = 8$

$(2^3)^2 \rightarrow 3\cdot2 = 6$

$2^3 \cdot 2^3$

$(2\cdot2\cdot2)\cdot(2\cdot2\cdot2)$

2^6

$2^2 3^2 = (2\cdot3)^2$

$(2\cdot2)\cdot(3\cdot3)$

$4 \cdot 9 = 36$

$2\cdot3 = (6)^2 = 36$

no zero real number: a and b
integers n and m

1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$
4. $a^0 = 1$
5. $a^{-n} = \frac{1}{a^n}$

Patterns in Exponents

⋮	⋮
5^4	625
5^3	125
5^2	25
5^1	5
5^0	1
5^{-1}	$\frac{1}{5}$
5^{-2}	$\frac{1}{25}$
5^{-3}	$\frac{1}{125}$
5^{-4}	$\frac{1}{625}$

Lesson

find the approximate square roots by iterative processes

$\sqrt{5}$ approximate to the nearest hundredths

$\sqrt{5}$ is between $\sqrt{4}$ and $\sqrt{9}$

$2^2 = 4$ and $\sqrt{4} = 2$

$3^2 = 9$ and $\sqrt{9} = 3$

2 2.1 2.2 | 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3

$\sqrt{4}$ $\quad \sqrt{5}$ $\quad \sqrt{9}$

x	x^2
2.0	4
2.1	4.41
2.2	4.84
2.3	5.29

Real Numbers, Scientific Notation, & Exponents

Real Numbers

Rational

$3\frac{1}{2}$ $\sqrt[3]{27}$ 4^{-2}

Integers
-2, -1, 0, 1, 2 ....

Whole #'s
0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ....

Natural #'s
1, 2, 3, 4 ....

Numbers that can be exactly named, like on a number line.

Irrational

ϕ = Phi - Golden mean
$\phi \approx 1.61803$ ... ellipsis
$\sqrt{2} \approx 1.41421...$
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$\sqrt{5} \approx 2.23606...$
e = Euler's constant
$e \approx 2.71828...$
π = Pi
$\pi \approx 3.14159...$

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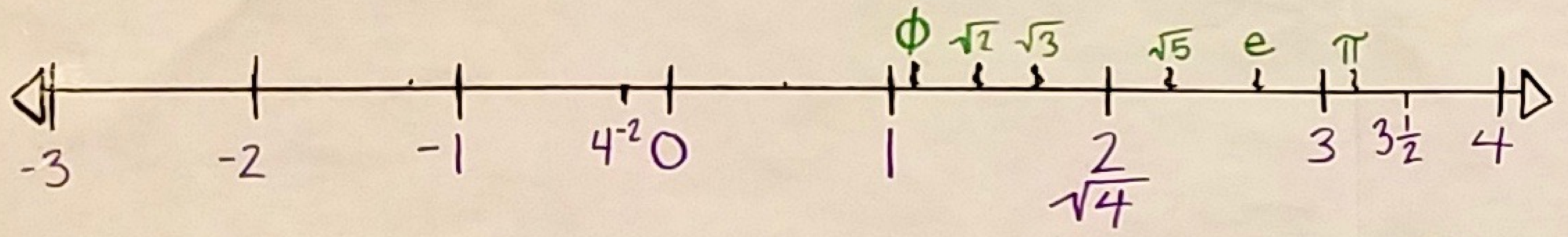

Standards and Mathematical Patterns

- We will attend to precision in knowing rational and irrational numbers.
  $3\frac{1}{2}, 0, -3$ — $\sqrt{2}, \phi, e$
- We will reason abstractly and quantitatively when using scientific notation (5×10^4) to measure very large and very small numbers.
- We will evaluate square roots ($\sqrt{4} = 2$) of perfect squares and cubed roots of perfect cubes by looking for and making use of structures.

Scientific Notation

Used to show very large and small numbers

1000	100	10	1		0.1	0.01	0.001
10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}
3,	4	2	6	.	4	5	6

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0.0000067	→	6.7×10^{-6}
7.30,000,000	←	7.3×10^8
.000045	←	4.5×10^{-5}

Expanded form

$\boxed{3,426.456}$

$3 \times 10^3 + 4 \times 10^2 + 2 \times 10^1 + 6 \times 10^0 + 4 \times 10^{-1} + 5 \times 10^{-2} + 6 \times 10^{-3}$

We know...	Inquiry: real numbers, Scientific Notation, Exponents	We want to know...
- Has to do with math - Scientific Notation has to do with science - They are all about numbers		- Why with exponents do you have to multiply the numbers like 2×2×2? (Exponent says how many times to use the number in a multiplication. (book)) - What is scientific notation? An easier way to write very large or small numbers. (lesson 1.4) - What are real numbers? All numbers on the number line, like positive and negative, integers, and rational numbers, square roots, cube roots, etc. (mathwords.com)

Exponents

Exponential forms

Perfect Squares x^2	Square roots $\sqrt{x}$	Perfect Cubes x^3	Cubed roots $\sqrt[3]{x}$
$1^2 = 1$	$\sqrt{1} = 1$	$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$	$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^2 = 9$	$\sqrt{9} = 3$	$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^2 = 16$	$\sqrt{16} = 4$	$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^2 = 25$	$\sqrt{25} = 5$	$5^3 = 125$	$\sqrt[3]{125} = 5$
$6^2 = 36$	$\sqrt{36} = 6$	$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^2 = 49$	$\sqrt{49} = 7$	$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^2 = 64$	$\sqrt{64} = 8$	$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^2 = 81$	$\sqrt{81} = 9$	$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^2 = 100$	$\sqrt{100} = 10$	$10^3 = 1000$	$\sqrt[3]{1000} = 10$

Patterns

⋮	⋮
5^4	625
5^3	125
5^2	25
5^1	5
5^0	1
5^{-1}	$\frac{1}{5}$
5^{-2}	$\frac{1}{25}$
5^{-3}	$\frac{1}{125}$
⋮	⋮

Integer Exponent Properties

Model	Proof	Rule
$2^3 \cdot 2^5 = 2^8$	$\underbrace{2 \cdot 2 \cdot 2}_{3} \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{5}$	$x^3 \cdot x^5 = x^{3+5} = x^8$
$(3^2)^3 = 3^6$	$(3^2)(3^2)(3^2)$	$(x^2)^3 = x^{2 \cdot 3} = x^6$
$2^3 \cdot 3^3 = 6^3$	$(8 \cdot 27) = 216$ $6 \cdot 6 \cdot 6 = 216$	$x^3 \cdot y^3 = xy^3$
$5^0 = 1$	$5^0 = 5^1 \div 5^1 = 5^{1-1} = 5^0 = 1$	$x^0 = 1$
$6^{-2} = \frac{1}{6^2}$	$1 \div 6 \div 6 = 0.028$	$x^{-n} = \frac{1}{x^n}$