Case Study: 5th Grade Unit on Division and Fractions
by Melissa Hancock and Mandy Wedel—Hillsboro School District, Oregon

Introduction

We are Melissa Hancock and Mandy Wedel, and this case study represents our work as two math coaches from different elementary schools, working in fifth-grade classrooms, in Oregon's Hillsboro School District. The district was searching for ways to bring language to the forefront of mathematics. Achievement Inspired Mathematics for Scaffolding Student Success (AIM4S$^{3™}$) was welcomed to the Hillsboro School District as a means to increase intentional scaffolding and sheltering in math instruction. Latinos in Hillsboro show significantly lower achievement when compared to their Caucasian counterparts. In the last 2 school years, 26.1% of Latino students in the district met or exceeded the proficiency level on the Smarter Balanced Assessment Consortium (SBAC), Oregon's state mathematics assessment, while 57.4% of Caucasian students met or exceeded proficiency on the same assessment. In addition, only 18.4% of English learners met or exceeded the proficiency level on the state math assessment over the same 2 years.

Setting

Melissa Hancock is in her 14th year of teaching at Reedville Elementary School and has been a math leader in the district for the past 7 years. This is her first experience teaching fifth grade. She began the year with 14 students and then, due to a staffing change, 15 more students joined the class in January. Twenty-three of the students were boys and 8 were girls. There were 6 special education students, and 22 of her students were identified as English learners based on their Oregon English Language Proficiency Assessment scores.

Melissa reviews a fraction chant with students. The chant was processed with students using highlighting and sketching words to be clarified and adding pictures for comprehensibility.

Mandy uses the Kagan strategy Numbered Heads to choose a student to report out for the team. This validates the importance of each student's voice and increases student engagement.

Four years ago, Reedville Elementary School was struggling. Low state test scores in the 2011-12 school year had earned the school a Federal Title I designation of "Focus," meaning they were ranked in the bottom 5%-15% of high poverty schools in Oregon. With additional support from the state, Reedville ramped up its interventions for students who needed extra assistance, including after-school tutoring. The following year, the school was ranked a Level 3 under the state's new rating system, indicating they were performing on par with most other schools in Oregon.

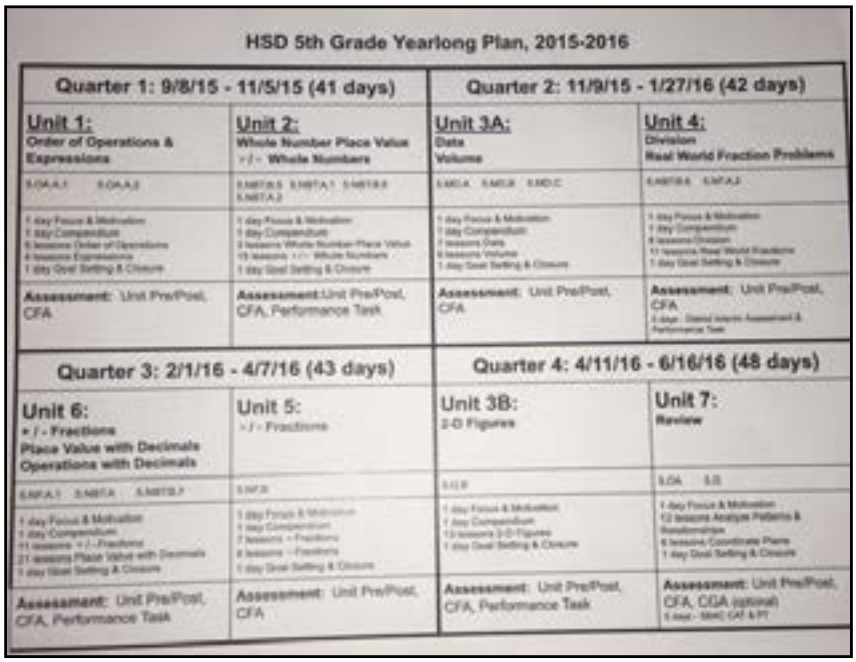
Mandy Wedel is also in her 14th year of teaching in the Hillsboro School District. She taught in the classroom for 12 years at Indian Hills Elementary in the fifth and sixth grades. Two years ago she took a new position as a math coach at Lincoln Street Elementary, a Title I school. Lincoln Street's student population is approximately 73% Hispanic and 23% Caucasian. To qualify for the free or reduced lunch program, a family's income must be 130% below the poverty line, which was $15,171 in 2015. Currently 79% of students at Lincoln Street qualify for free lunch. At the school, 52% of students are identified as English learners and 14% of the students qualify for special education. Students at Lincoln Street demonstrate a very low rate of proficiency on state assessments. In 2015-2016, 32% of males and 17% of females met or exceeded the proficiency level on the SBAC math assessment. As of the 2016-2017 school year, Lincoln Street has been designated as a trauma-

sensitive school and is currently rated a Level 1 on the state's 1 to 5 rating scale (retrieved from *http://public-schools.startclass.com/l/108437/Lincoln-Street-Elementary-School*).

Mandy worked with a class of 28 fifth-grade students. There were 14 boys and 14 girls. Five students were enrolled in the ESL program, and 18 of them reported Spanish as their home language. There were two students receiving special education services.

Planning for the Year

When generating our year-long plan, we began with the district math planner. This planner was created by the district math leadership team consisting of teachers and math leaders representing all schools in the district. The purpose of the planner was to align the district-adopted program (*Bridges in Mathematics*, Edition 1) with the Common Core State Standards in Mathematics.


HSD 5th Grade Yearlong Plan, 2015-2016

Quarter 1: 9/8/15 - 11/5/15 (41 days)		Quarter 2: 11/9/15 - 1/27/16 (42 days)	
Unit 1: Order of Operations & Expressions	**Unit 2:** Whole Number Place Value + / - Whole Numbers	**Unit 3A:** Data Volume	**Unit 4:** Division Real World Fraction Problems
Assessment: Unit Pre/Post, CFA	Assessment: Unit Pre/Post, CFA, Performance Task	Assessment: Unit Pre/Post, CFA	Assessment: Unit Pre/Post, CFA
Quarter 3: 2/1/16 - 4/7/16 (43 days)		**Quarter 4: 4/11/16 - 6/16/16 (48 days)**	
Unit 6: x / ÷ Fractions Place Value with Decimals Operations with Decimals	**Unit 5:** x / ÷ Fractions	**Unit 3B:** 2-D Figures	**Unit 7:** Review
Assessment: Unit Pre/Post, CFA, Performance Task	Assessment: Unit Pre/Post, CFA	Assessment: Unit Pre/Post, CFA, Performance Task	Assessment: Unit Pre/Post, CFA, CGA (optional)

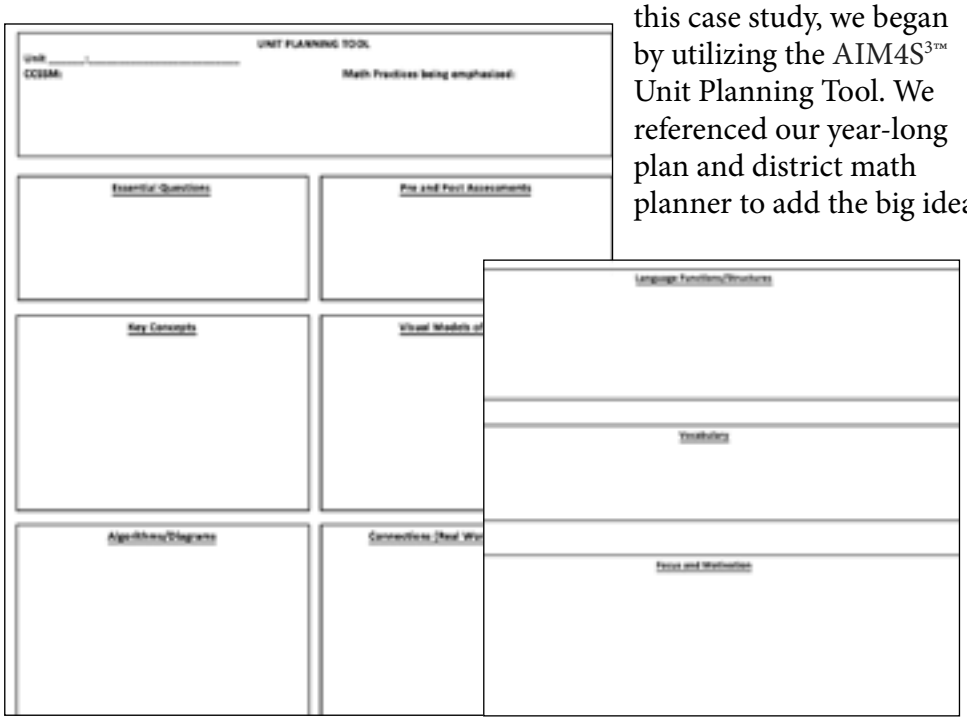
Our first task was to analyze the district math planner for a scope and sequence of the fifth-grade math standards. We listed the units, recorded the standards taught in each unit, and identified the big ideas for each unit. Then we compared the list to the Common Core State Standards to be sure that all standards were taught throughout the year.

To backwards map, we looked at the district calendar and the number of instructional days in each quarter. There are 8 units in the math planner—approximately 2 units per quarter. However, as we dug into the program to identify the number of lessons provided for each topic in the unit we discovered that some units are longer than others, which might require carryover from one quarter to the next. We also noted the different assessments, including pre/post, district Common Formative Assessments (CFAs) and performance tasks, that were provided to support each unit. This information allowed us to make notes on topics to include in the concept frame of the Compendium for each unit.

We felt energized as we analyzed our final year-long plan and adjusted pacing to include the components of the AIM4S$^{3\text{™}}$ framework. We had sufficient time to teach not only all of the fifth-grade standards, but we were also able to incorporate the framework components to support all students with the language and content of mathematics. The process of backwards planning allowed us to start the year with a big picture which helped with unit planning throughout the year. This also confirmed that the AIM4S$^{3\text{™}}$ framework supports the math planner provided to all teachers in Hillsboro School District.

Planning for the Unit

When it came time to plan Unit 4, which is the focus of this case study, we began by utilizing the AIM4S$^{3\text{™}}$ Unit Planning Tool. We referenced our year-long plan and district math planner to add the big ideas


UNIT PLANNING TOOL

Unit: _____

CCSSM: Math Practices being emphasized:

Essential Questions

Pre and Post Assessments

Key Concepts

Visual Models of

Language Functions/Structures

Vocabulary

Algorithms/Diagrams

Connections (Real Wo

Focus and Motivation

and standards to the planning tool. In addition, we took the district unit assessments prior to beginning lesson planning in order to be certain they aligned to standards and would give us the information we needed to inform instructional decisions. The focus for Unit 4 is division and real world fraction problems. The AIM4S^{3}™ framework provided an opportunity to connect these two concepts. We were able to show this connection on the Compendium, provide additional Focus and Motivation activities, and add a supplemental task to the unit lessons. Despite being at different schools, we found the opportunity to plan collaboratively invaluable.

Mandy challenges students to identify the big idea or take away of the growth mindset video they are about to watch.

In working with her students during the first half of the school year, Mandy noticed that the group as a whole lacked self-confidence in math and showed evidence of a fixed mindset in regards to their own math potential. Melissa faced similar challenges when her class size doubled. We both felt it necessary to begin by building Positive Classroom Culture through working with students on a growth mindset. We used Jo Boaler's *Positive Classroom Norms* and showed the *Mathematical Mindset* videos from *youcubed.org*'s *Week of Inspirational Math*. These conversations helped to set a tone of positive growth mindset, which continued to develop in both classrooms over the course of the unit. We both observed students taking risks and being open to attempting new strategies while learning from each other.

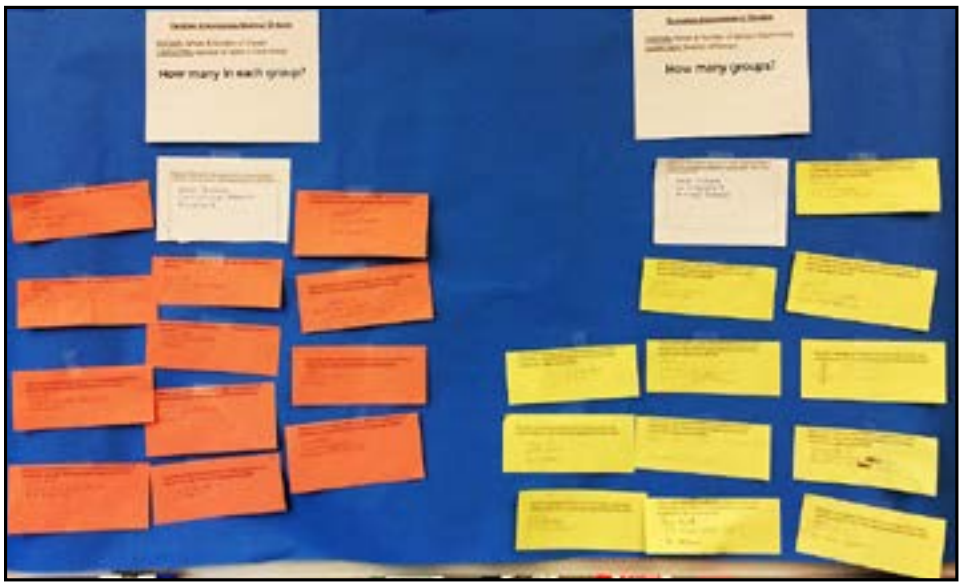
Case Study Components

Focus and Motivation

We used Focus and Motivation activities at the beginning and throughout the unit, considering different learning styles when selecting activities. These activities consisted of videos, literature, games, and hands-on learning.

We incorporated a variety of videos throughout the unit to build students' interest and deepen their conceptual understanding. One BrainPop® video (*https://www.brainpop.com/math/numbersandoperations/division/*) was used to build a common understanding of division and to create a shared definition of division as the splitting of large numbers into equal groups of smaller numbers. Students were given the option to take notes on the video if they felt it would help them to process the information.

We intentionally spent time working on real-world problems that required students to develop an understanding of division as finding the number of items in each group (partitive) *or* finding the number of groups (quotative). We shared a short video from PBS Learning Media (*https://opb.pbslearningmedia.org/resource/mgbh.math.ns.twodiv/two-kinds-of-division/#.WSR_tOsrIdU*) that demonstrates the difference between the two types of division using manipulatives and cookies as a context. After watching this video, students had an opportunity to explore different word problems and sort them into the two categories of division. This experience helped to not only solidify their understanding of division, but also to make connections between the math they were learning in class and its real-world applications.



Students analyzed word problems to identify if they were looking for the number of groups or the number in each group. This activity deepened students' understanding of division.

As we worked through a few of our unit lessons and were exploring division with larger numbers (NBT.B.6), we wanted to make the connection between division and multiplication. We showed a video from LearnZillion (*https://learnzillion.com/lesson_plans/8786-divide-4-digit-dividends-by-2-digit-divisors-by-using-a-rectangular-array*) that demonstrates division using base ten area pieces to break a larger problem into smaller parts using multiplication. This supported student learning by connecting to prior knowledge of multiplication and linking a familiar strategy to a new concept.

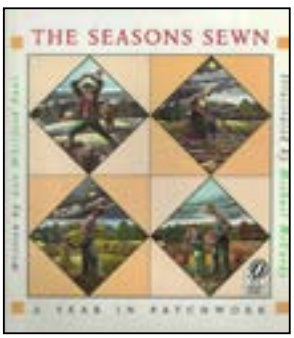
During the Listen and Respond activity, students in Mandy's classroom reflected on how the book connected to their understanding of division.

Another Focus and Motivation activity that we used at the beginning of the division part of our unit was a Listen and Respond with the book, *The Great Divide: A Mathematical Marathon* (Dodds, 2005). Students were encouraged to respond using words, numbers, pictures, or any other way that made sense to them. The benefit of this activity is not only to introduce a new math concept in a way that lowers the affective filter through literature, but it is also a chance for us to gather informal data on the students' prior knowledge of division. We have found that when numbers are the focus of the activity, students will lean on a standard algorithm, even if it is not fully understood. The Listen and Respond strategy puts the focus on the content, which allowed students to use pictures to model with mathematics.

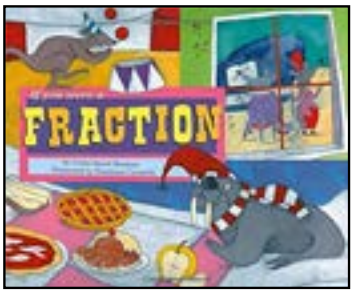
Throughout the unit, students played the games *Divisibility Rules* and *Lowest Remainder Wins* which are embedded into the daily lessons in *Bridges*. These games were introduced whole group and were gradually released to students. First the whole class played against the teacher, then they played in groups, and finally with a partner. Reinforcing standards through a game allows students to practice the skill in an engaging environment that is non-threatening.

This time also allowed us the opportunity to touch base with students who, through informal observation, had been identified as needing additional support. When it came time to transition from division to fractions during our unit, we gave the students a "low-floor, high-ceiling" task that demonstrated the connection between the two concepts. Low-floor, high-ceiling tasks have an access point for all students, but allow for students to increase complexity of the task based on their skill level. We asked, "There are 7 brownies and 4 friends. How many brownies would each friend get?" After making sense of the problem and persevering through the task, some students were able to offer their connection of the division problem of 7 divided by 4 and the answer that each friend received 7/4 of a brownie. This provided a natural transition to our exploration of fraction problems in the real world.

Because fifth grade is the last year in the CCSS Number and Operations - Fractions standards progressions and because we noticed gaps in conceptual understanding of standards from previous grade levels, we spent extra time with Focus and Motivation activities on this topic of fractions. We began by sharing two YouTube videos of images on fractions in real-life *(https://www.youtube.com/watch?v=5AVjBFP4MRg, https://www.youtube.com/watch?v=pTbCbMOmVFc)*. This prompted a discussion about where we find fractions in our daily lives. Students had the opportunity to share what they saw in the videos and expand their ideas to include examples from their personal experiences.



Next, we shared two picture books with the students. We chose to read the first book, *The Seasons Sewn* (Paul, 1996), as a traditional read aloud because the pictures related to real-life examples and the content



connected well with the demographics of our school. The next day we used *If You Were a Fraction* (Shaskan, 2008) as a Listen and Respond using the same protocol explained previously.

In order to make sense of unit fractions conceptually, we had all students make their own set of fraction bars out of construction paper. They created halves, fourths, eighths, and sixteenths. Then they used these fraction bars to play a game called *Race to Two and Back* (NF.A. 1 & 2). In this game, the students needed to add and subtract fractions with different denominators. They each had their fraction bars to support finding equivalent fractions and working through the operations. With these experiences, they were ready to start the unit lessons and apply this understanding to real-world problems.

Compendium

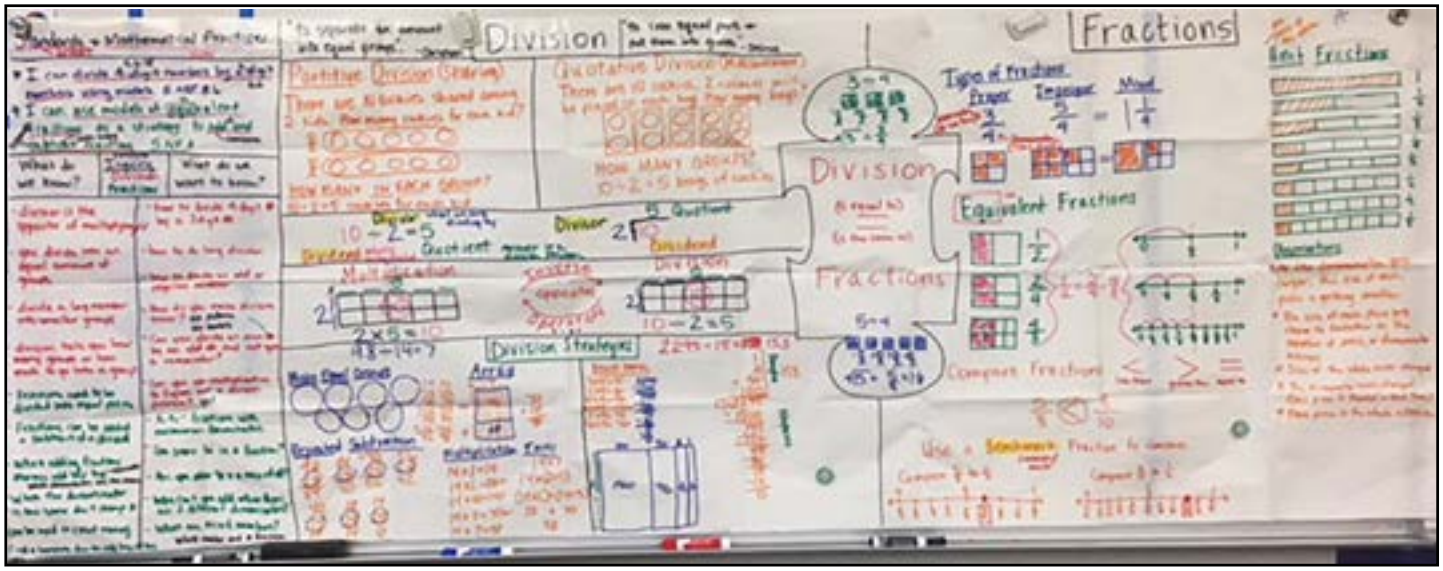
We planned the Compendium collaboratively as a part of unit planning. However, as the unit unfolded and we responded to student needs in each classroom, the final products looked quite different.

The Compendium was introduced on the second day of the unit in both classrooms, starting with the "Division" half of the Compendium. Students had been working on the meaning of division in a previous unit, so they began this unit with some common prior knowledge. Only the division standard and mathematical practice were introduced and processed during this time.

Students used fraction bars to support their conceptual understanding of equivalent fractions.

Next, we moved on to the inquiry chart. Due to the work already done in the school year, students made many accurate contributions to the left side of the inquiry chart on what they already knew about division. At this point Melissa realized that she recorded student voice without leaving space for processing. She chose to add additional paper to the bottom of her Compendium to make space for further student contributions.

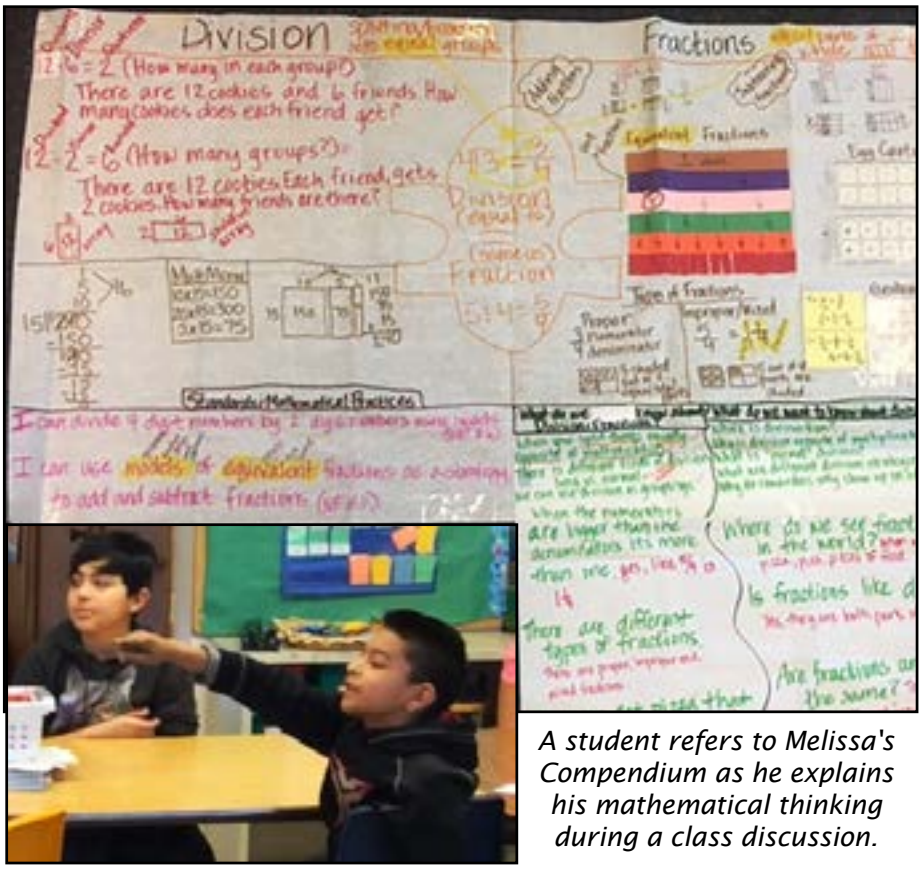
Prior to beginning any unit lessons, both of us front-loaded the part of the concept frame describing the two types of division using real-world context. Mandy chose to define these two



Mandy's class Compendium supports their unit on division and fractions. See pages 10 and 11 to compare how each teacher developed her Compendium to meet her students' specific needs.

types of division for her students, however it was not an expectation that they memorize the name of each type. During the inquiry in Mandy's classroom, students mentioned the connection between multiplication and division, so she decided to add a section showing the inverse relationship of the two operations.

The "Division Strategies" section of the concept frame was not added until needed to support unit lessons. Mandy chose to add strategies that students used to make sense of a word problem using two-digit numbers. All these strategies had been introduced in previous units. Melissa chose to only include the new strategy to support the standard being assessed because her students could reference other strategies on a past Compendium.



A student refers to Melissa's Compendium as he explains his mathematical thinking during a class discussion.

An important concept in this unit is the connection between division and fractions. To represent this relationship, we chose to use a puzzle piece. Within the puzzle piece we added vocabulary, example equations, and visuals in Mandy's case. A similar rollout occurred with the "Fraction" side of the Compendium. The title, student-friendly standards, and mathematical practices were shared and processed. There was time to engage in inquiry about fractions.

Since this was the first time students had worked with fractions in fifth grade, and based on pre-assessment data, we knew we needed to anchor some key fraction concepts for students to build upon. We both included "types of fractions" on the Compendium. Based on Mandy's data, she felt the need to include "equivalent fractions", "comparing fractions", and "unit fractions." Each of these anchors are standards from previous grade levels and would be important for making sense of real-world fraction problems. Melissa added visual models that were used in unit lessons to make sense of the standard being assessed. She chose to make them interactive by using packaging tape so that they could be written on and erased.

Throughout the unit, we kept the students' ideas and questions about division and fractions from the inquiry at the forefront while we were teaching. As clarifications were discovered or questions were answered, we facilitated a discussion with students and added this thinking to the inquiry chart. We each spent time as a part of the unit Closure to go back and review ideas that were still lingering. Additional student responses were added to the inquiry chart at this time.

We felt that using the Compendium with this group of diverse learners allowed all students access to the standards. The Compendium supported students in becoming independent learners and seeking answers to their own questions. Having the standards and mathematical practices on the Compendium helped us focus our work. Students knew the end goal and how the lessons were aligning to the standards.

Unit Lesson

Both classrooms had 60 minutes scheduled for math instruction. A typical math lesson began with a Focus and Motivation activity, whole group instruction, or adding to the Compendium, if needed. The remaining time during the lesson included small group and independent work. Within the unit, students engaged in hands-on activities including games,

Three Reads Strategy

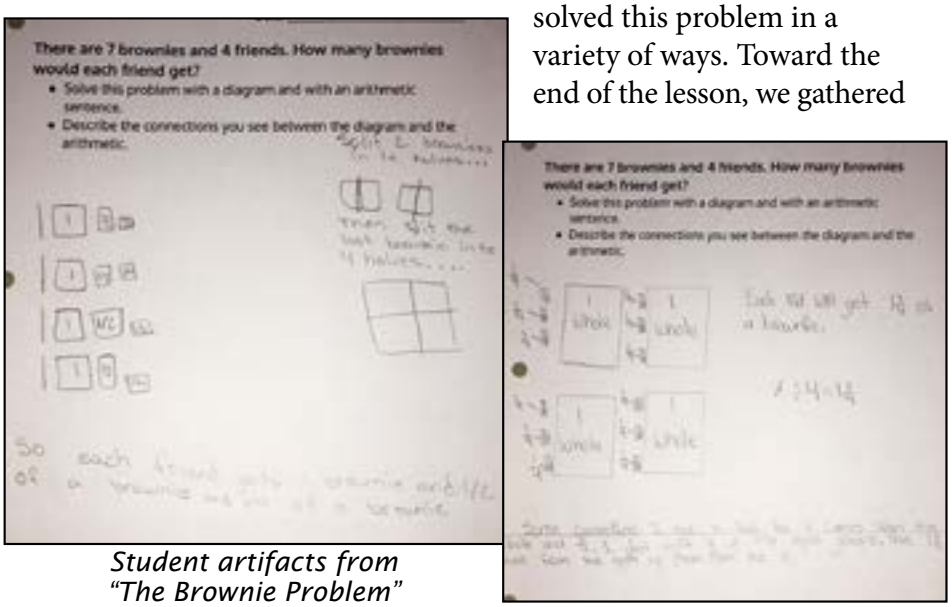
1st read	2nd read	3rd read
Ana spent ___ of her money on a pack of pens, ___ of her money on a pack of markers, and ___ of her money on a pack of pencils.	Ana spent ⅓ of her money on a pack of pens, ½ of her money on a pack of markers, and ⅛ of her money on a pack of pencils.	Ana spent ⅓ of her money on a pack of pens, ½ of her money on a pack of markers, and ⅛ of her money on a pack of pencils. What fraction of her money is left?

manipulatives, and real-world application. Common Formative Assessments (CFA) were embedded throughout the unit to gather data that guided instructional decisions. As a part of the master schedule at Melissa's school, paraprofessional support was embedded into every math block. Melissa's intervention block came at the end of her whole group instruction time and was used to support grade-level standards at the student's level. Groups were formed based on data gathered from CFAs, which were given approximately every two weeks. This allowed for flexible grouping, and she was better able to reach the needs of all her students.

Mandy did not have the luxury of this support in the master schedule at her school. Instead, after whole-group instruction and using a Continuous Feedback strategy of student self-assessment of understanding, she would call students to the front of the room to provide support. Some students were asked to join her and others were given the option if they felt the need. Students were encouraged to support each other by asking and answering each other's questions. Even though extra adult support would have been ideal, this structure provided Mandy the opportunity to meet the individual needs of students.

Throughout this unit students were required to model with mathematics as they applied their understanding of division and fractions to real world problems. To shelter the language and scaffold the content, we utilized the *Three Reads Strategy* for problem solving. During the first read of the problem, numbers and values are removed as well as the question, in order to focus on the context and language in the problem. For the second read, the numbers and values are added back in, but the question is still left out. The purpose of this read is to apply the meaning of each number or value to the context of the story. Next, for the third read, students write mathematical questions that could be answered using the given context and values. Finally, the teacher gives them the question she would like them to answer. At this point students have a deep understanding of the context, language, and values in the problem and how all of these parts are related. Breaking down problems in this way helped all students to model mathematics, to make sense of the problems, and to persevere in solving them. We found that the sheltering of language and scaffolding of content provided by this strategy led to success for many of our students.

At this point in our unit we transitioned from division to fractions, with the low-floor, high-ceiling open-ended task, "The Brownie Problem", described in the section on Focus and Motivation. Neither of us provided any guidance or suggestions on how to begin. Students were encouraged to work in collaborative groups. There were a variety of tools available, including tiles, graph paper, construction paper, and scissors. Students were engaged in the task and began discussing strategies right away. Teams solved this problem in a variety of ways. Toward the end of the lesson, we gathered



Student artifacts from "The Brownie Problem"

around one table and used construction paper and scissors to model how 7 brownies would be shared with 4 friends. When all students agreed that each of the 4 friends would receive 1¾ brownies, or 7/4 brownies, the teacher asked about the equation that would describe this situation. It was then that a student offered that 7 divided by 4 = 7/4. This was the connection we were hoping for between division and fractions, so we were ready to add this puzzle piece to the Compendium.

We were very excited to share our students' experiences with The Brownie Problem with each other. We both agreed that the time spent prior to this unit on building a growth mindset helped students to take risks while working in their collaborative groups. After students worked independently on the problem, each class came together to discuss it. We saw students using their adaptive reasoning as they made sense of the task in a new way or for the first time as they heard ideas from other students. Both of us felt that the time spent frontloading mindset and content knowledge allowed for success during unit lessons.

Individual reflection and goal setting was a vital part of our positive classroom culture and supported our work around growth mindset.

Closure and Goal Setting

We made sure to take the time for Closure and Goal Setting activities. We know that it is often overlooked due to the time that it takes, but we found the payoff in student ownership to be invaluable.

After students completed the unit post-test, both of us selected student responses for whole class analysis. Sometimes this student work produced the correct answers; sometimes it did not. Some samples had well-constructed responses, while others were limited. The goals for this analysis were to critique the reasoning of others in a safe environment and then to compare and reflect on their own responses. Students were able to recognize strengths and misconceptions in other students' work and noticed growth in their own responses.

The next step in our unit closure was to ask students to reflect on their personal growth from their pre-assessment to their post-assessment responses. In Melissa's class, student journals were used on a regular basis to record reflections and personal math goals. The classroom where Mandy was teaching did not use math journals, so their reflection and goals happened on notebook paper instead. Students were asked to look beyond the total points, and instead, focus on the quality and quantity of responses. We asked each student to share at least one area that they were proud of and one area that they would like to improve. This reflection and goal setting built on all of the work we had previously done around growth mindset and encouraged students to take ownership of their own learning.

As a part of our final celebration we returned to one of our Listen and Respond books, *If You Were a Fraction*, and shared it as a read aloud. Students compared their original responses with the illustrations in the book. Students enjoyed seeing the illustrations and making connections to their learning.

During our reflection on the conclusion of the unit, we both identified processing of the inquiry chart on the Compendium as an area of improvement. Each of them has evidence of some processing, but we both feel that a more specific focus on this would be more meaningful for students. We also hypothesized that as students become more familiar with the inquiry portion of the Compendium, their responses and questions will elicit higher level thinking that will lead to deeper investigation of the mathematical content.

Final Reflection

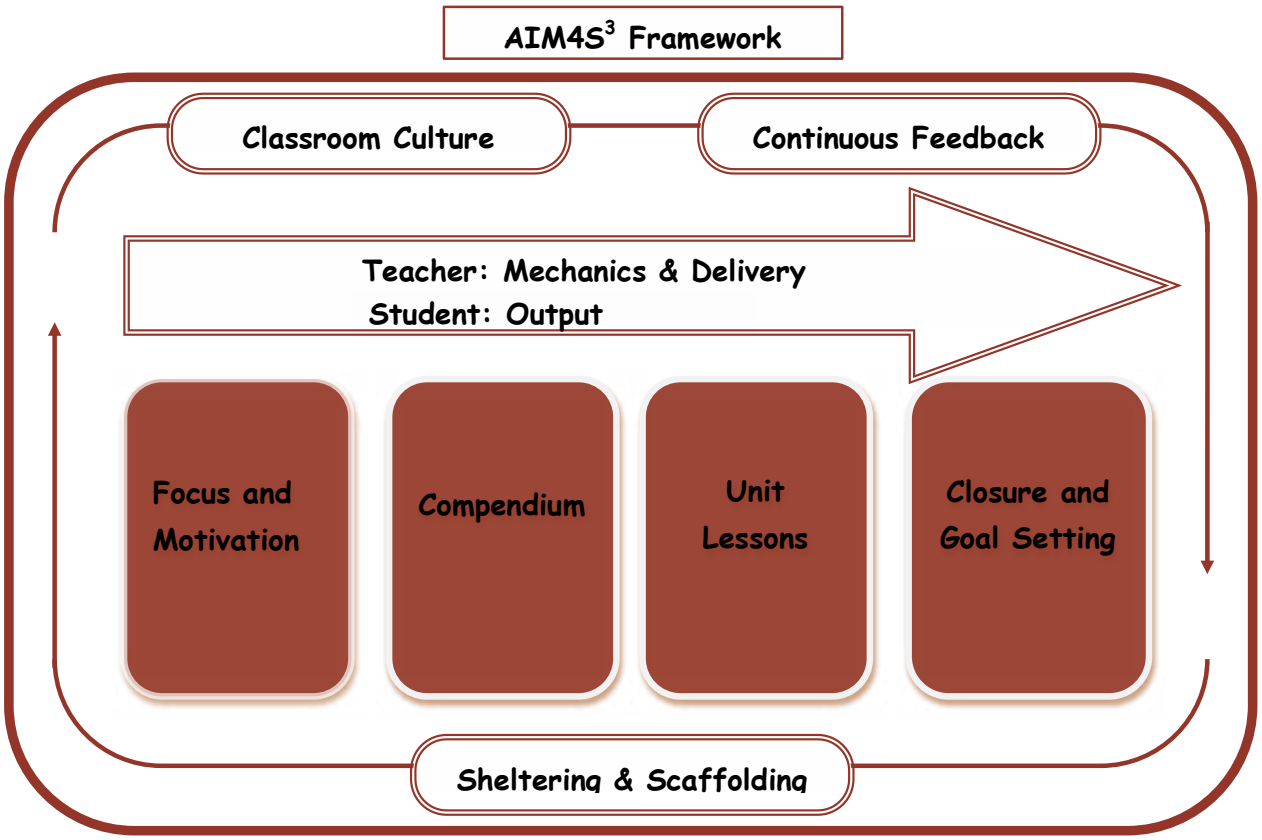
We have found the Unit Planning Tool to be a crucial resource for backwards planning for successful unit implementation. It continues to support us in seeing how the unit focuses on standards and fits into the big picture of our fifth-grade year-long plan. Without

the Unit Planning Tool, our instruction would not have been as focused and the framework components may not have been addressed adequately.

Embedding the Key Instructional Principles into daily instruction made a positive influence on the improved growth mindset that we have seen in our students. The purposeful and engaging lessons that were planned as a part of this unit led to students taking more risks as mathematicians. As students experienced success because of their risk-taking, we saw improvements in their self-confidence. Our hope is that over time these behaviors result in a more student-centered math learning community.

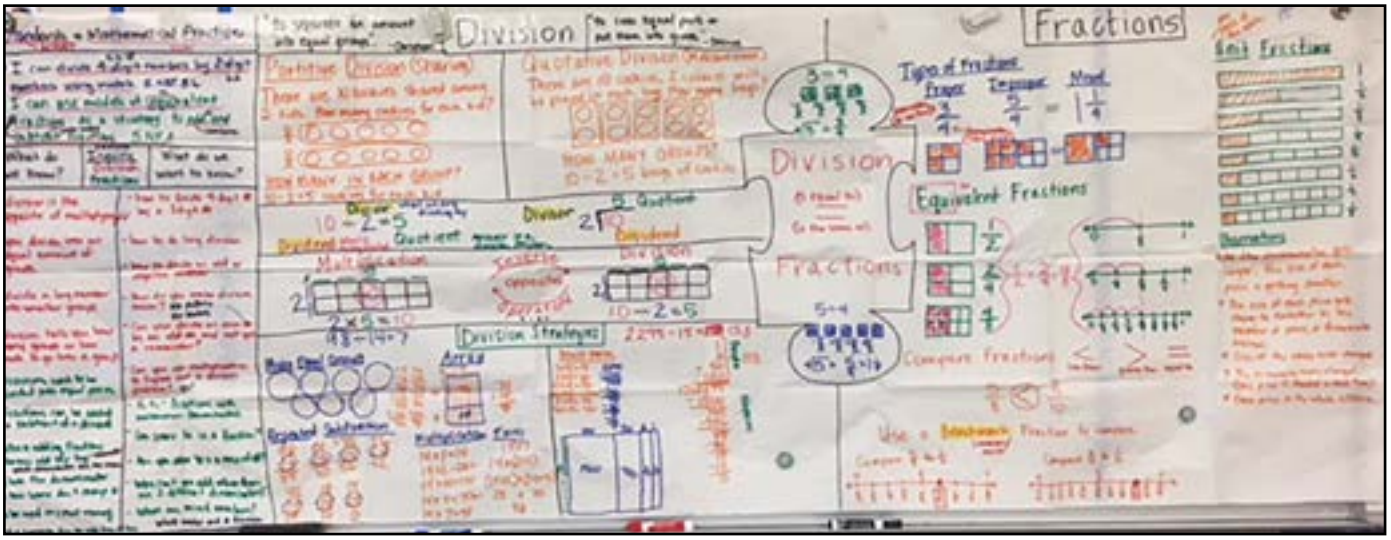
Melissa (left) and Mandy (right) found planning collaboratively to be a powerful tool for deepening their instruction.

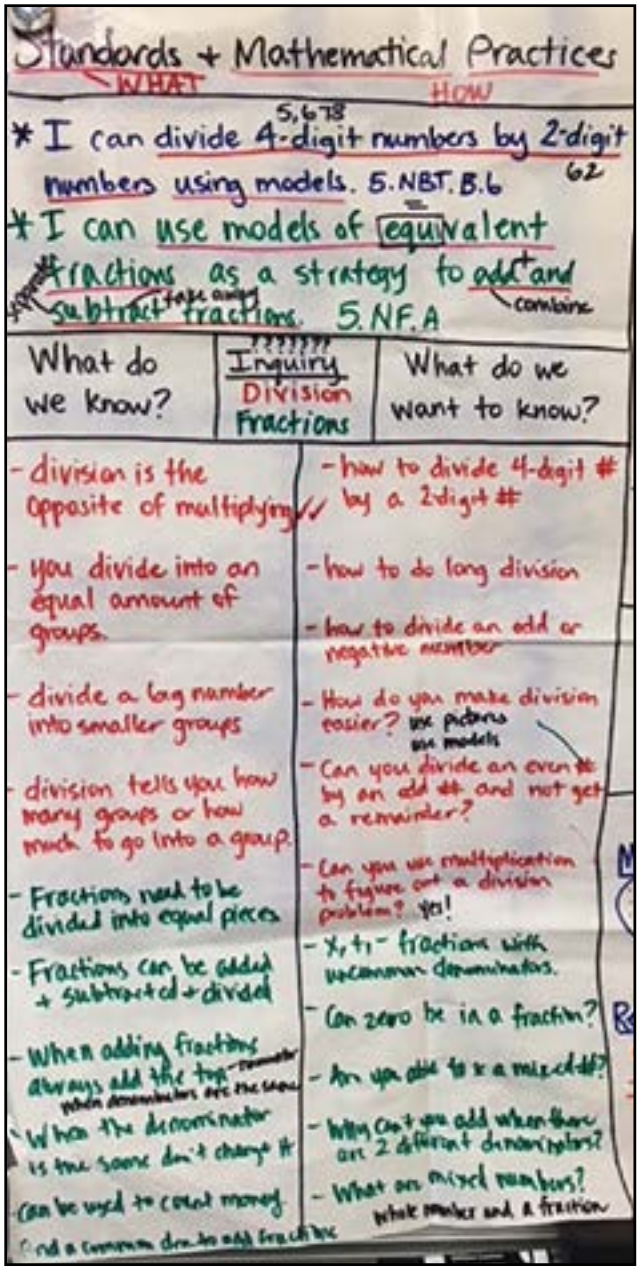
Having a thought partner throughout the planning, implementation, and reflection process of this entire AIM4S^{3}™ framework has been vital. From the early stages of planning, as we were working on embedding all fifth-grade math standards into our year-long plan and then making specific plans for each of the framework components in this unit, we were able to share and elaborate on each other's ideas. During the unit lessons, we often touched base and checked in on student progress, leading to modifications in our daily plans. Finally, having a reflective partner who shared in a similar experience allowed us each to make adjustments to our teaching for the next unit and year.

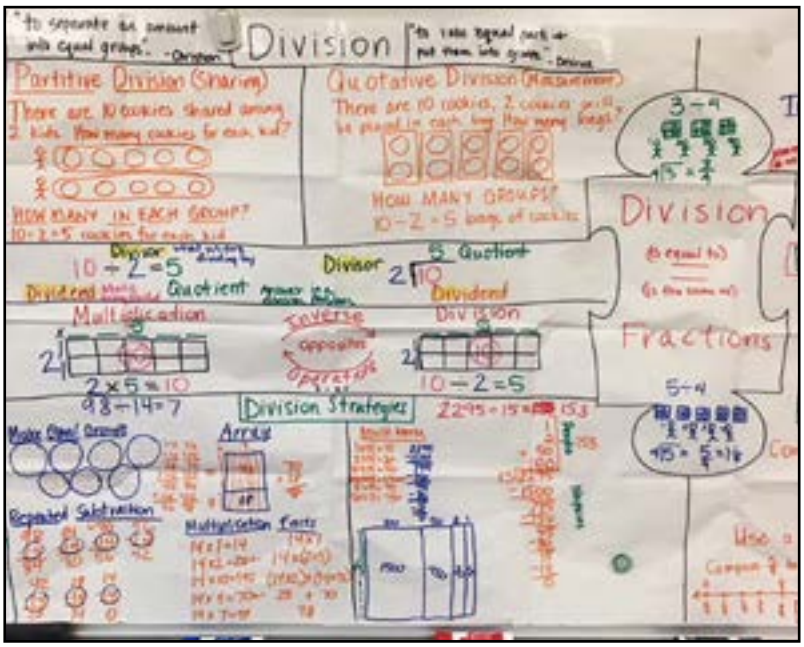


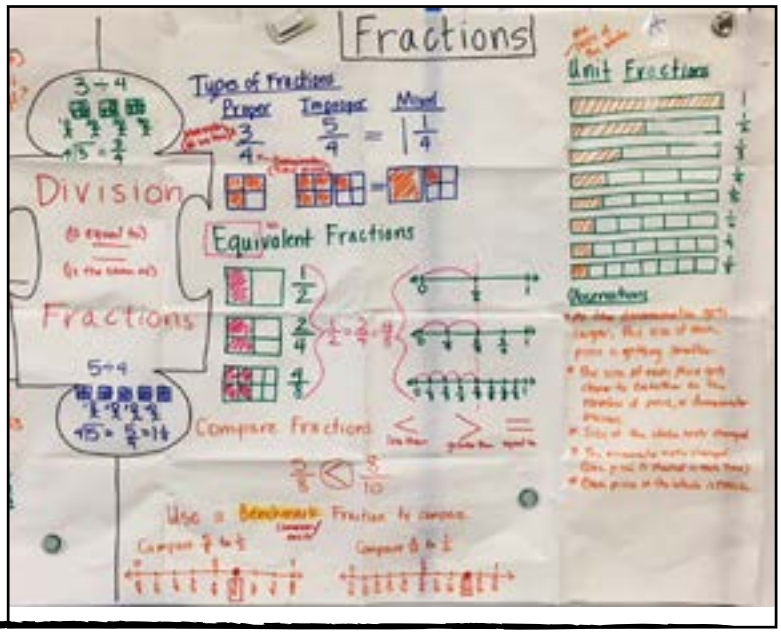
Appendix

Mandy's Compendium

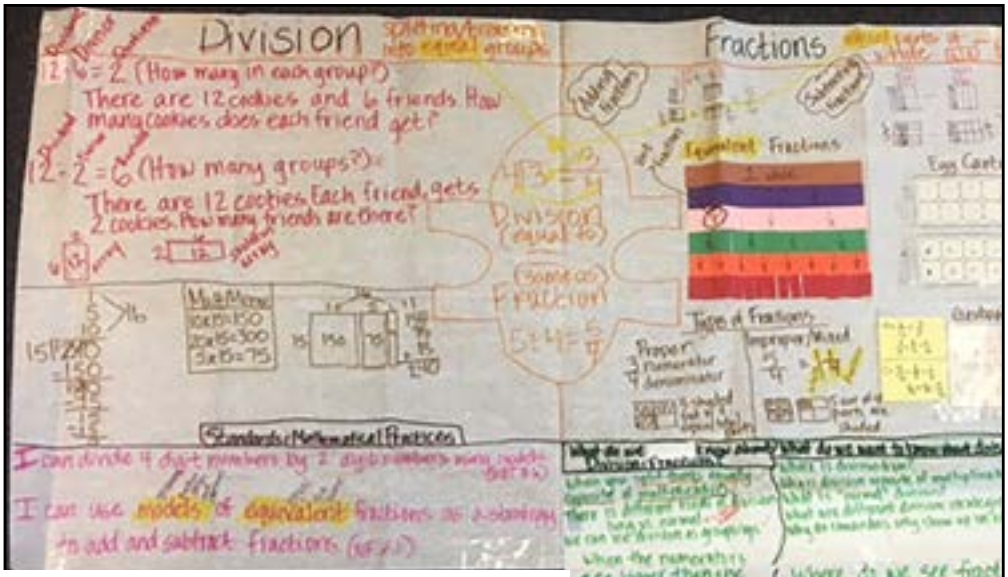


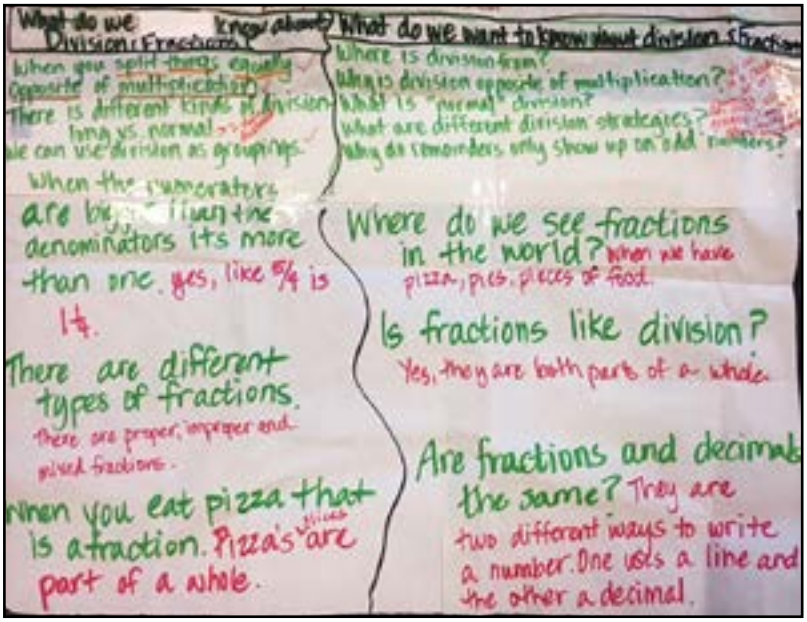


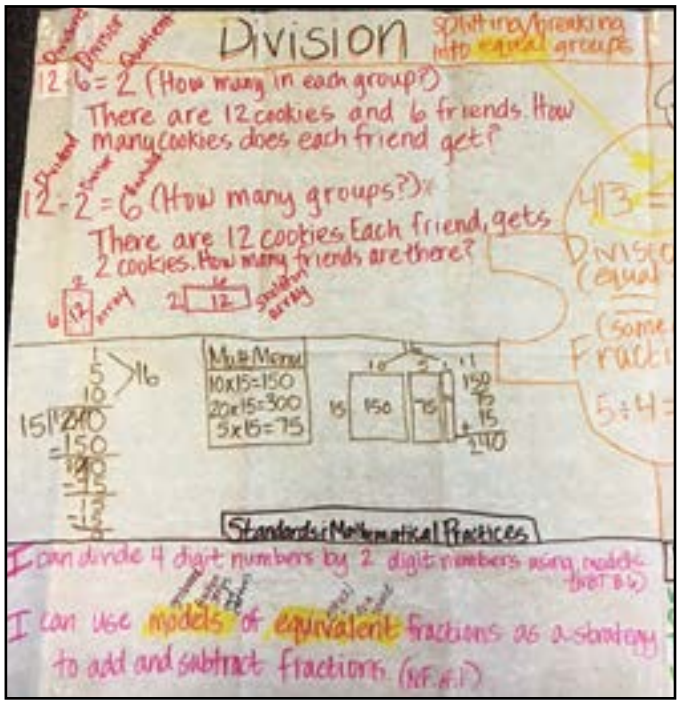


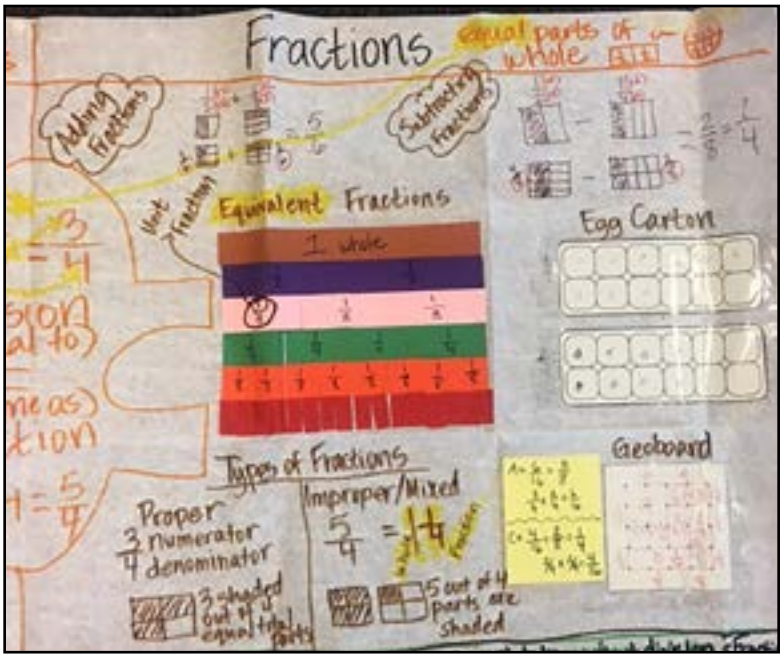


Melissa's Compendium









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